College of Engineering
UNIVERSITY OF HAWAI'I

HCAC Hawaii Center for Advanced Communications

Computational Techniques in Electromagnetics EE-693J

Magdy F Iskander

Spring 2010

Lecture #1

PDF

Objective:

The course covers popular computational methods (integral equation-Method of Moments, Differential equations-finite difference, and finite element methods), routinely used to solve practical engineering problems.

The objective is to provide an introductory level of understanding of these methods, compare them, and provide guidelines for their application to solve problems that are formulated in terms of either integral or differential equations.

Why learn more than one technique?

Selection of the appropriate computational method for a specific engineering problem is key in achieving successful and accurate solution

Course Procedure:

- At the begging of the course, a **brief review** will be provided for the following topics:
  - Differential and integral forms of Maxwell's equations
  - Examples of EM Engineering Problems Formulated in terms of Differential and Integral Equations
  - Difference between this computational techniques course and other numerical methods courses
  - Detection and avenues to test and avoid generation of ill-conditioned systems of equations
  - Eigen value problems, and difference from regular system of simultaneous equations
- Then there will be lectures describing each of the three popular computational methods in electromagnetics
- In each case, examples will be provided to illustrate the solution procedure

Course Procedure (continued):

- For each of the three computational techniques (Finite Difference, Method of Moments, and Finite Element), students will be asked to write a program that solves simple problem in electromagnetics.
- Students can use any available programming tools, including Matlab, C, LabView, etc.
- These programming assignments will count towards the mid term exams
- A final exam will also be scheduled and this will focus on the comparative aspects of these computational techniques and their use in solving practical engineering problems
- Grade distribution will be based on:

Mid-term assignments, three @25% each	75%
Final Exam	25%
Total	100%

PDF

College of ENGINEERING
UNIVERSITY OF HAWAI'I

HCAC Hawaii Center for Advanced Communications

Course Procedure (continued):

- Text and course material:
  - This course will be taught jointly at the University of Hawaii in the US and Tsinghua University in China. Course material will be available on the Internet at the Hawaii Center for Advanced Communications (HCAC) Website
  - Considerable amount of the course material is described in the paper, "*A New Course on Computational Methods in Electromagnetics," IEEE Transactions on Education,* vol. 31, pp.101-111, 1988.

PAPER

  - For the introductory material on the Finite Difference and Method of Moments, reference will be made to the material available in chapter 4 of the text, *Electromagnetics Fields and waves*, by Iskander, Waveland Press, 2000
  - Besides the above listed resources, additional class material will be provided and made available on the HCAC website.

Blank

PDF

ications

清华大学
中国 北京 100084

Tsinghua University
Beijing 100084, China

Department of Electronic Engineering
Tsinghua University, Beijing 100084

电子工程系

Professor Magdy F. Iskander
Hawaii Center for Advance Communications
College of Engineering, University of Hawaii
1680 East West Rd., Post 201
Honolulu, HI 96822
USA

Dear Prof. Magdy F. Iskander

We are pleased to invite you to teach a distance learning version of your course on "Computational Electromagnetic" for the engineering graduate students at Tsinghua University. The course will be for two credit hours and will require students to complete the required projects and obtain letter grads for their work.

Based on our students' initial experience with the short course on "New Progresses on Antennas and Propagation Research" you recently presented at our university, we are expecting that these activities will be mutually beneficial and will help with growing a productive and successful implementation of the recently signed MOU between the Hawaii Center for Advance Communications at University of Hawaii and the State Key Laboratory on Microwave and Digital Communications at Tsinghua University. We are appreciate your continue interest and we wish you and the collaborating colleagues in Tsinghua University continue success.

Prof. Yidong Huang
Deputy Chair of Dept. of Electronic Engineering

PDF

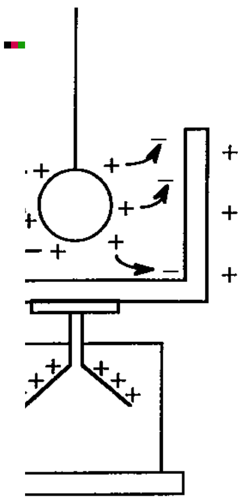
Section 1, Review items

1.1 Integral and differential forms of Maxwell's equations

- These four equations are fundamental to our study in this course. We will solve electromagnetics (EM) engineering problems by formulating them in terms of integral or differential equations. So this will be based on these four Maxwell's equations and the boundary conditions associated with each problem.
- So what are these four equations?
- They are four mathematical equations that relate the vector electric **E** and magnetic **B** fields (bold letters) to their vector current **J** and charge ρ sources
- They are all (except one term in Ampere's law) are based on experimental observations, actually simple experiments, so here they are (next slide)

- **Gauss' law for electric field**: This one is based on the Ice pail experiment conducted by Faraday

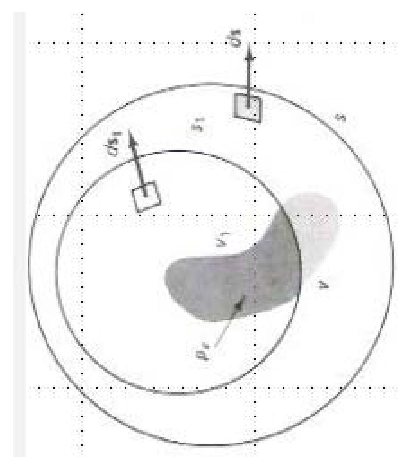
$$\oint_s \epsilon_o \mathbf{E} \cdot d\mathbf{s} = \int_v \rho_v \, dv$$



The electric flux density $= \dfrac{(\psi_e = Q)}{\text{area of spherical surface}} = \dfrac{Q}{4\pi r^2}$, $\bar{E} = \dfrac{Q}{4\pi\varepsilon_o r^2}\,\bar{a}_r$

$\varepsilon_o \bar{E} = \dfrac{Q}{4\pi r^2}\,\bar{a}_r$ = electric flux density in $\bar{a}_r$ direction

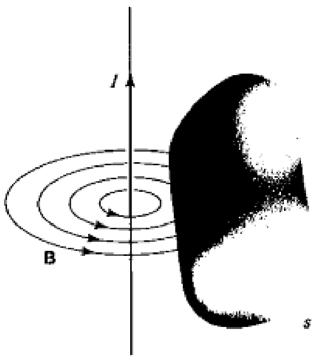
- This equation simply says that the total electric flux (ε_o**E** is the electric flux density) out flowing from a closed surface S is equal to the total charge enclosed inside the surface or within the volume v enclosed by the surface.



- **Gauss' Law for magnetic field**
  - This is similar to that for the electric field with the exception that there is no isolated single magnetic charge and hence the right hand side of the equation is equal to zero, Hence

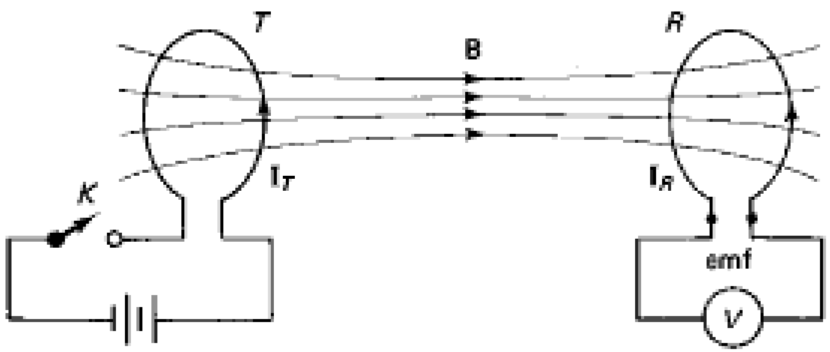
$$\oint_s \mathbf{B} \cdot d\mathbf{s} = 0$$

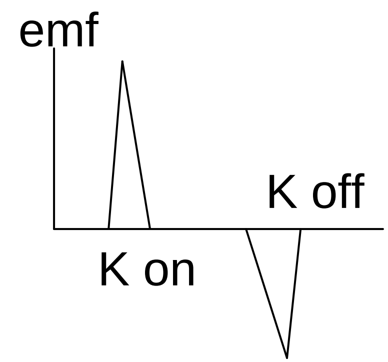
  - **B** is the magnetic flux density, so with the integration we calculate the total magnetic flux out flowing from the closed surface S. The zero value on the right hand side of the equation does not indicate the absence of magnetic field but instead that the number of magnetic flux lines entering the surface equal to the number leaving the surface
  - Two main observations may be noted from this equation
    - As mentioned before, there is no single magnetic charge that can be isolated and enclosed inside the surface S. Magnetic poles (north and south poles) exist in pairs
    - It shows that magnetic flux lines are closed lines (i.e. number going into the surface equal to the number out (see figure)



PDF

- Faraday's Law





  - This law resulted from the simple experiment shown above. The induced *emf* in the receiving coil R was detected only during the opening and closing of the switch K.

$$emf = \oint_c \mathbf{E} \cdot d\ell = -\frac{d}{dt} \int_s \mathbf{B} \cdot d\mathbf{s}$$

B, it was discovered that
ng the magnetic flux, and
o the magnetic flux.

PDF

- Faraday's law (continued)

$$emf = \oint_c \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d}{dt}\int_s \mathbf{B} \cdot d\mathbf{s}$$

•This law can be summarized as follows:

- The induced *emf* resulted from the time varying ($\frac{d}{dt}$) magnetic field that is crossing the area s of the receiving loop R $\int_s \mathbf{B} \cdot d\mathbf{s}$
- The electric field term in the middle of the equation shows that the induced *emf* resulted from the induced electric field around the magnetic field. This electric field circulated charges around the contour c in the receiving loop, and this what caused the observed induced *emf.*
- The critical observation here is related to the fact that time varying magnetic field generates or is associated with and electric field

PDF

- **Amp**
  - A

$$\oint_c \frac{\mathbf{B}}{\mu_o} \cdot d\ell = \int_s \mathbf{J} \cdot d\mathbf{s}$$

which simply relates the circulating magnetic field to the current source producing it.

  - Maxwell found that this law violates the laws of physic and specifically the law the low of conservation of charge

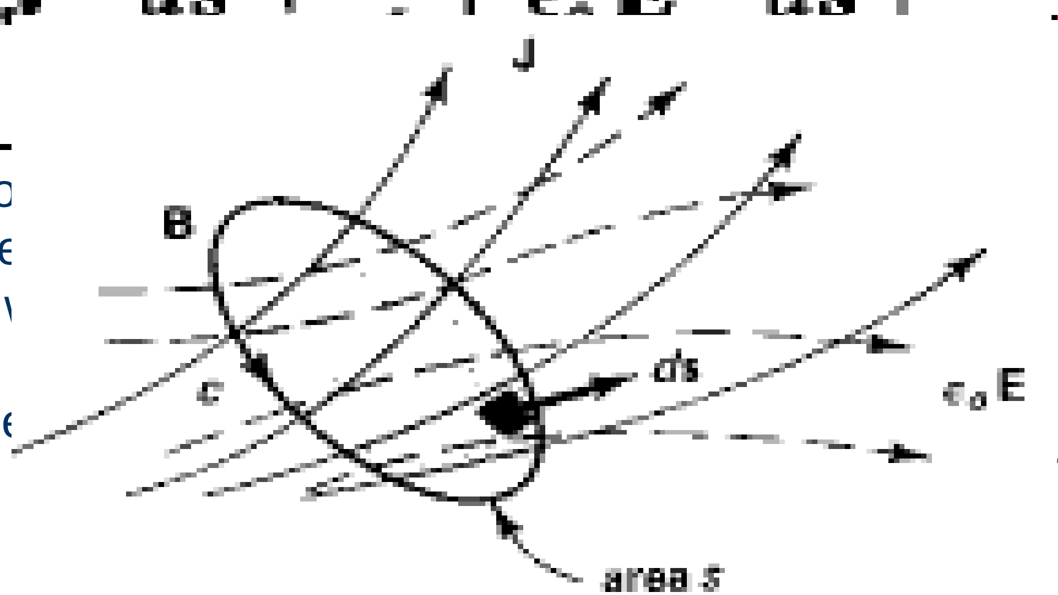
$$\oint_c \frac{\mathbf{B}}{\mu_o} \cdot d\ell = \int_s \mathbf{J} \cdot d\mathbf{s} + \frac{d}{dt}\int_s \epsilon_o \mathbf{E} \cdot d\mathbf{s}$$

PDF

$$\oint_c \frac{\mathbf{B}}{\mu_o} \cdot d\ell = \int_s \mathbf{J} \cdot ds + \frac{d}{dt}\int_s \epsilon_o \mathbf{E} \cdot ds$$

– Based on this new fo ... erating the circulating magnetic fie... conventional currents ...

and also time varying e...



– This mathematically ... n as together with Faraday's law we now have time varying magnetic field generates electric field (Faraday's law) and time varying electric field generates magnetic field (the new term in Ampere's law) and hence the **phenomena of wave propagation**.

PDF

Summary of Maxwell's Equations

	INTEGRAL FORM	DIFFERENTIAL FORM
Gauss's law of electric field	$\oint_s \mathbf{D} \cdot d\mathbf{s} = \int_v \rho_v \, dv$	$\nabla \cdot \mathbf{D} = \rho_v$
Gauss's law of magnetic field	$\oint_s \mathbf{B} \cdot d\mathbf{s} = 0$	$\nabla \cdot \mathbf{B} = 0$
Faraday's law	$\oint_c \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d}{dt}\int_s \mathbf{B} \cdot d\mathbf{s}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
Ampere's law	$\oint_c \mathbf{H} \cdot d\boldsymbol{\ell} = \int_s \mathbf{J} \cdot d\mathbf{s} + \int_s \sigma \mathbf{E} \cdot d\mathbf{s} + \frac{d}{dt}\int_s \mathbf{D} \cdot d\mathbf{s}$	$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} + \sigma \mathbf{E}$

where $\mathbf{D} = \epsilon_o \epsilon_r \mathbf{E}$ and $\mathbf{B} = \mu_o \mu_r \mathbf{H}$

TABLE 2.1 SUMMARY OF MAXWELL'S EQUATIONS IN DIFFERENTIAL FORM

1. Gauss's Law for Electric Field

$$\text{Div}\,\epsilon_o\mathbf{E} = \nabla\cdot\epsilon_o\mathbf{E} \triangleq \lim_{\Delta v\to 0}\left[\frac{\oint_{\Delta s}\epsilon\mathbf{E}\cdot d\mathbf{s}}{\Delta v}\right] = \frac{\epsilon_o}{h_1h_2h_3}\left[\frac{\partial(E_1h_2h_3)}{\partial u_1}+\frac{\partial(E_2h_1h_3)}{\partial u_2}+\frac{\partial(E_3h_1h_2)}{\partial u_3}\right] = \rho_s$$

2. Gauss's Law for Magnetic Field

$$\text{Div}\,\mathbf{B} = \nabla\cdot\mathbf{B} \triangleq \lim_{\Delta v\to 0}\left[\frac{\oint_{\Delta s}\mathbf{B}\cdot d\mathbf{s}}{\Delta v}\right] = \frac{1}{h_1h_2h_3}\left[\frac{\partial(B_1h_2h_3)}{\partial u_1}+\frac{\partial(B_2h_1h_3)}{\partial u_2}+\frac{\partial(B_3h_1h_2)}{\partial u_3}\right] = 0,$$

Concept | Vector analysis compact symbolization "shorthand notation" | Defining equation "scalar" | Mathematical relationship resulting from application of defining equation in the generalized curvilinear coordinate system | Physical quantity at the point P_o at which we evaluate the divergence of the vector field

Concept

Defining equation Mathematical Relation

Physical quantity at a point

3. Faraday's Law

$$\text{Curl}\,\mathbf{E} = \nabla\times\mathbf{E} \triangleq \sum_{k=u_1,u_2,u_3}\mathbf{a}_k \lim_{\Delta s(k)\to 0}\left[\frac{\oint_{C(k)}\mathbf{E}\cdot d\mathbf{l}}{\Delta s(k)}\right] = \begin{vmatrix}\frac{\mathbf{a}_1}{h_2h_3} & \frac{\mathbf{a}_2}{h_1h_3} & \frac{\mathbf{a}_3}{h_1h_2}\\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3}\\ h_1E_1 & h_2E_1 & h_3E_2\end{vmatrix} = -\frac{\partial\mathbf{B}}{\partial t}$$

4. Ampere's Law

$$\text{Curl}\,\frac{\mathbf{B}}{\mu_o} = \nabla\times\frac{\mathbf{B}}{\mu_o} \triangleq \sum_{k=u_1,u_2,u_3}\mathbf{a}_k \lim_{\Delta s(k)\to 0}\left[\frac{\oint_{C(k)}\frac{\mathbf{B}}{\mu_o}\cdot d\mathbf{l}}{\Delta s(k)}\right] = \begin{vmatrix}\frac{\mathbf{a}_1}{h_2h_3} & \frac{\mathbf{a}_2}{h_1h_3} & \frac{\mathbf{a}_3}{h_1h_2}\\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3}\\ h_1\frac{\mathbf{B}_1}{\mu_o} & h_2\frac{\mathbf{B}_2}{\mu_o} & h_3\frac{\mathbf{B}_3}{\mu_o}\end{vmatrix} = \mathbf{J}+\frac{\partial\epsilon_o\mathbf{E}}{\partial t}$$

Concept | Vector analysis compact symbolization "shorthand notation" | Defining equation (vector form) | Mathematical relationship resulting from application of defining equation in the generalized curvilinear coordinate system | Physical quantity at point P_o

Concept Vector analysis

Defining equation Mathematical Relation

Physical quantity at a point

PDF

Simple example of combined use of Maxwell's equations

Wave equation in source free region, Iskander, pp-150-151

$$\nabla \cdot \epsilon_o \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \frac{\mathbf{B}}{\mu_o} = \frac{\partial \epsilon_o \mathbf{E}}{\partial t}$$

$$\boxed{\nabla^2 \mathbf{E} - \mu_o \epsilon_o \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0}$$

$$\boxed{\nabla^2 \mathbf{B} - \mu_o \epsilon_o \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0}$$

PDF

Time-Harmonic Fields and Phasor Representation

(Iskander, 151-154)

$$\nabla \cdot \epsilon_o(\hat{\mathbf{E}}(\mathbf{r}))e^{j\omega t} = \hat{\rho}\, e^{j\omega t} \qquad \nabla \cdot (\epsilon_o\, \hat{\mathbf{E}}(\mathbf{r})) = \hat{\rho}$$

$$\nabla \cdot (\hat{\mathbf{B}}(\mathbf{r})\, e^{j\omega t}) = 0 \qquad \nabla \cdot \hat{\mathbf{B}}(\mathbf{r}) = 0$$

$$\nabla \times (\hat{\mathbf{E}}(\mathbf{r})\, e^{j\omega t}) = -j\omega(\hat{\mathbf{B}}(\mathbf{r})\, e^{j\omega t}) \qquad \nabla \times \hat{\mathbf{E}}(\mathbf{r}) = -j\omega\hat{\mathbf{B}}(\mathbf{r})$$

$$\nabla \times \left(\frac{\hat{\mathbf{B}}(\mathbf{r})\, e^{j\omega t}}{\mu_o}\right) = \hat{\mathbf{J}}(\mathbf{r})\, e^{j\omega t} + j\omega\epsilon_o(\hat{\mathbf{E}}(\mathbf{r})\, e^{j\omega t}) \qquad \nabla \times \frac{\hat{\mathbf{B}}(\mathbf{r})}{\mu_o} = \hat{\mathbf{J}}(\mathbf{r}) + j\omega\epsilon_o\, \hat{\mathbf{E}}(\mathbf{r})$$

$$\boxed{\nabla^2 \hat{\mathbf{E}} + \omega^2 \mu_o \epsilon_o \hat{\mathbf{E}} = 0} \qquad \boxed{\nabla^2 \hat{\mathbf{B}} + \omega^2 \mu_o \epsilon_o \hat{\mathbf{B}} = 0}$$

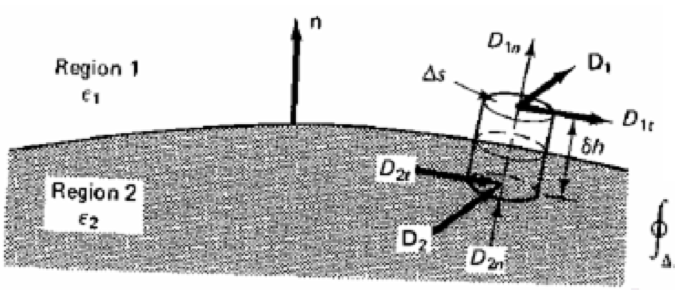
$$\mathbf{E}(\mathbf{r}, t) = Re(\hat{\mathbf{E}}(\mathbf{r})\, e^{j\omega t})$$

$$\mathbf{B}(\mathbf{r}, t) = Re(\hat{\mathbf{B}}(\mathbf{r})\, e^{j\omega t})$$

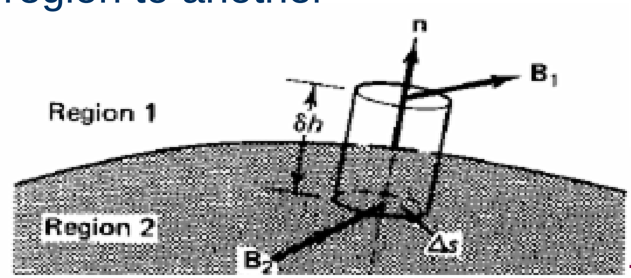
PDF

Formulation of an Electromagnetics Engineering Problem!..

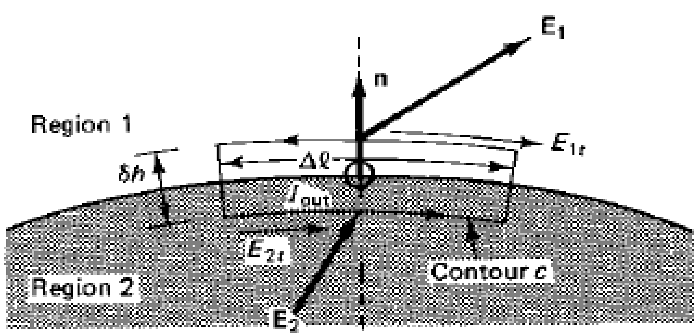
- Using Maxwell's equations to relate sources to induced fields in a given geometry
- Using boundary conditions
  - These are mathematical relations that describe the transitional properties of the electric and magnetic fields from one region to another



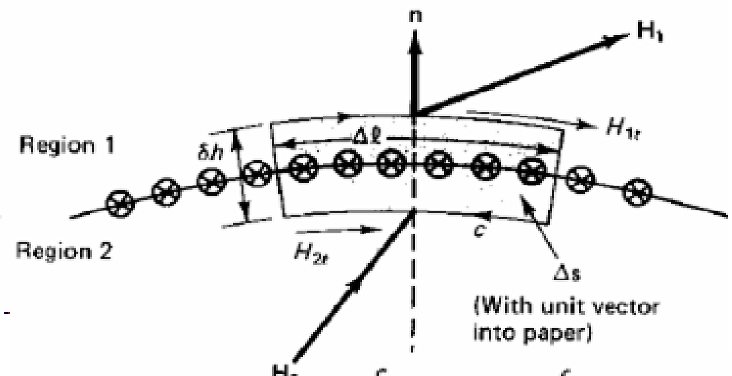
$$\oint_{\Delta s} \mathbf{D} \cdot d\mathbf{s} = \int_v \rho_v \, dv$$



$$\oint_s \mathbf{B} \cdot d\mathbf{s} = 0$$

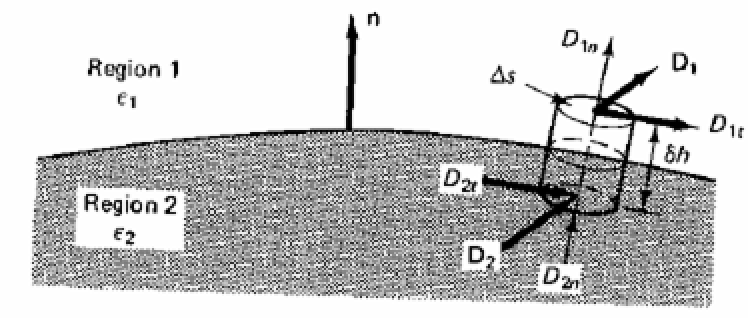


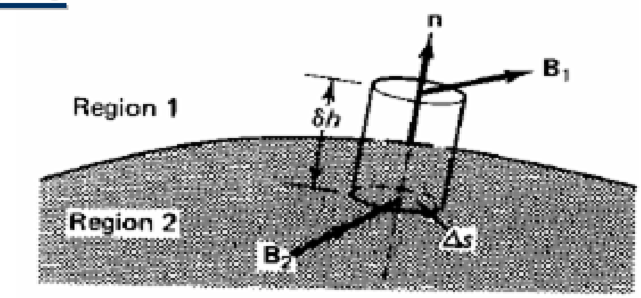
$$\oint_c \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d}{dt}\int_s \mathbf{B} \cdot d\mathbf{s}$$

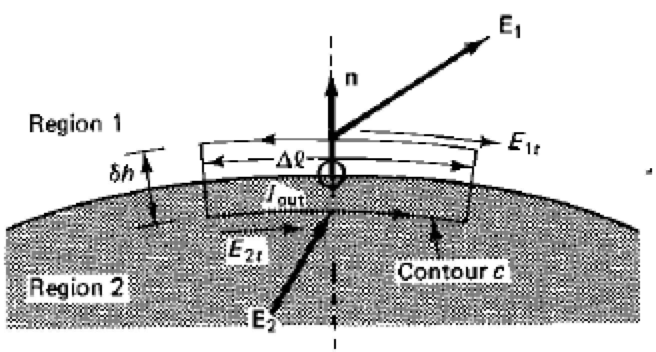


$$\oint_c \mathbf{H} \cdot d\boldsymbol{\ell} = \int_s \mathbf{J} \cdot d\mathbf{s} + \frac{d}{dt}\int_s \mathbf{D} \cdot d\mathbf{s}$$

E- and H-Fields Boundary Conditions

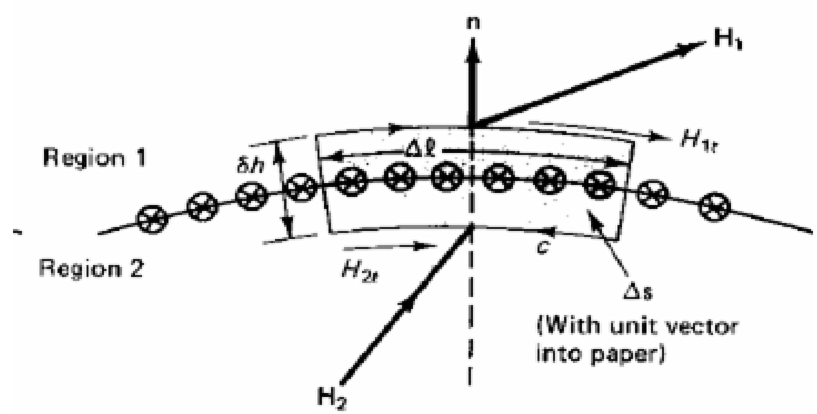






ELECTRIC FIELD BOUNDARY CONDITIONS

$$\mathbf{n} \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_v$$
$$\mathbf{n} \times (\mathbf{E}_1 - \mathbf{E}_2) = 0$$



BOUNDARY CONDITIONS

$$\mathbf{n} \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0$$
$$\mathbf{n} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$$

PDF

Question: why was it necessary to review Maxwell's equations and Boundary conditions?

Answer: Because we will use these equations together with the set of boundary conditions to derive mathematical relations that describe the behavior of EM engineering systems (transmission lines, waveguides, antennas, scattering objects, etc.). These mathematical relations may be in differential or integral forms

- If the geometry of the engineering problem is simple, analytical solution such as those based on separation of variables will be possible.
- If we are dealing with complex geometries and a variety of boundary conditions that need to be satisfied at different interfaces, we need to use computational methods to solve these problems
- This later case is the focus of our course, Computational Techniques in Electromagnetics..
- In the following slides we will derive some of these differential and integral equations that will be used to solve practical engineering problems.

PDF