College of Engineering
University of Hawai'i

Hawaii Center for
Advanced Communications

Computational Techniques in Electromagnetics
EE-693J

Spring 2010
Lecture #11

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1.2.3 Problems formulated in terms of integral equations:
Example 3: Capacitance of parallel plate capacitor

Why calculate capacitance of this simple geometry?

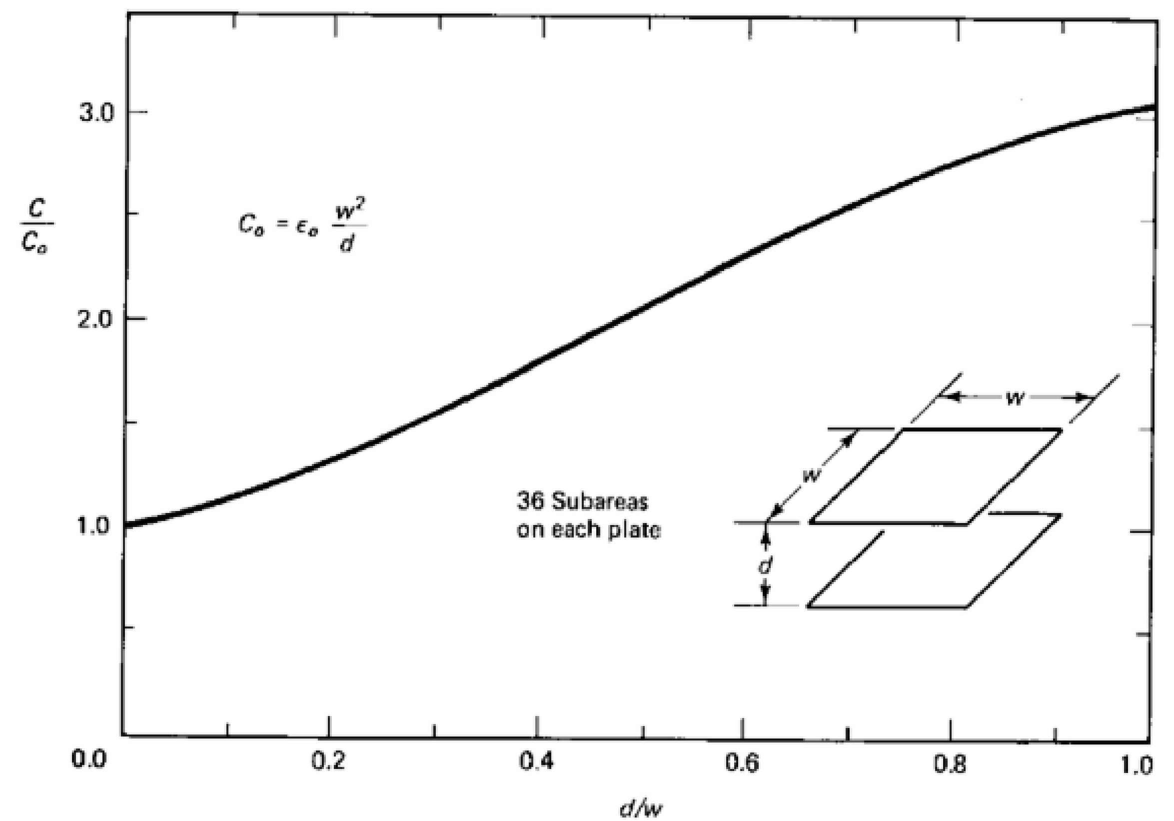
Answer:

- Available expression

$$C_o = \varepsilon_o A / d$$

Is valid for d<w, i.e. neglect fringing effects

- For d ≅ w, $C/C_o \approx 3$
- It may also be of interest if the plates are not exactly parallel, fringing fields effects



Problems formulated in terms of integral equations:
Example 3: Capacitance of parallel plate capacitor (continued)

$$C = Q/\Phi$$

Coulomb's law

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_o r^2}\mathbf{a}_r \;,\quad \Phi_a = \int_a^O \mathbf{E}\cdot d\ell$$

$$\Phi_a = \int_a^O \frac{Q}{4\pi\epsilon_o r^2}\mathbf{a}_r \cdot dr\,\mathbf{a}_r = \left[-\frac{Q}{4\pi\epsilon_o r}\right]_a^O = \frac{Q}{4\pi\epsilon_o r} - \frac{Q}{4\pi\epsilon_o r_o}$$

Zero reference potential at ∞ ,then

$$\Phi = \frac{Q}{4\pi\epsilon_o r}$$

For N discrete charges

$$\Phi = \sum_{k=1}^{N} \frac{Q_k}{4\pi\epsilon_o r_k}$$

For charge distribution ρ_v

$$\Phi = \int_v \frac{\rho_v\, dv}{4\pi\epsilon_o r}$$

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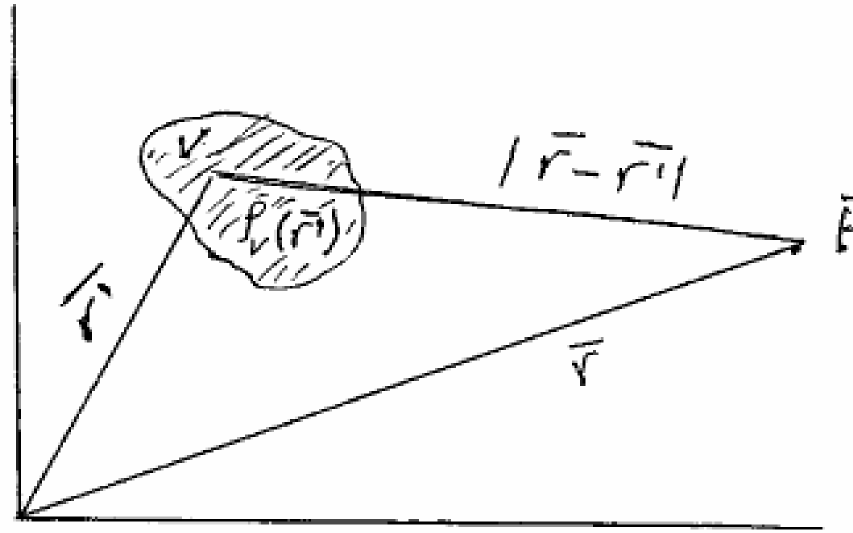
Example 3: Integral Equation Formulation

Integral equation for the potential $\Phi(\bar{r})$ is not a simple integration problem as the integrand $\rho(\bar{r}')$ is unknown

This integral equation will be solved for $\rho(\bar{r}')$ using the method of moments.

Once $\rho(\bar{r}')$ is obtained, capacitance will be calculated



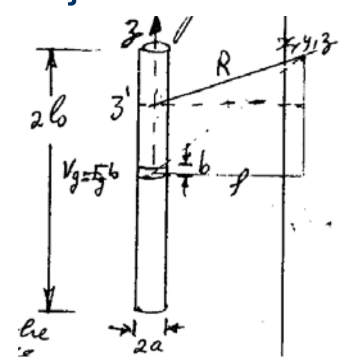
$\bar{r}'$ Source coordinate system
$\bar{r}$ observation coordinate system
$|\bar{r}-\bar{r}'|$ distance between source to observation point P

$$\Phi(\bar{r}) = \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho_v(\bar{r}')}{|\bar{r}-\bar{r}'|}\,dv'$$

1.2.4 Integral Equation formulation of Wire antenna Problems
Hallen's Integral equation

Another example that illustrates the use of Maxwell's equations and the EM boundary conditions is the integral formulation of radiation from wire antennas. Detailed derivation will be explained later, let us just see what it looks like

$$\int_{-l_0}^{l_0} J_z(z') \frac{e^{-jk_0R}}{R} dz' = -\frac{jV_g}{2\eta_0} \sin k_0|z| + C \cos k_0 z$$



Here Vg is the applied voltage in the transmitting port of the antenna of total length L. Once again, this is an integral equation with an unknown integrand J(z'), the current distribution along the wire antenna. Knowledge of the current distribution J(z') is very important in antenna analysis and here we have an integral equation that we need to solve to determine J(z'). In this course we will do this using the **Method of Moments**

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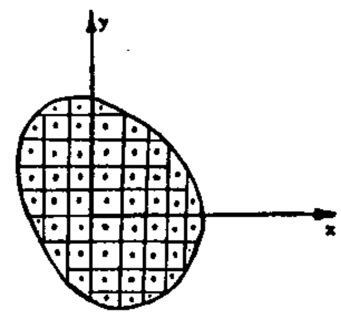
1.2.5 Two Dimensional Scattering by Dielectric Objects (TM case)
Scattering and Absorption by a Cross Section of a Human Body

This is yet another engineering problem of significant importance, including evaluation of biological effects associated with EM radiation, and also to help evaluate potential EM techniques for medical diagnostics and treatment.

For the 2D TM case, we will develop the following integral equation

$$E_z = E_z^i - \frac{k\eta}{4}\int_s j\omega(\epsilon - \epsilon_0)\, E_z H_0^{(2)}(k|\bar{\rho} - \bar{\rho}'|)\, ds.$$



The **unknown** induced electric field inside the dielectric object E_z, is once again inside the integral sign, and we will solve this integral equation using the **Method of Moments**

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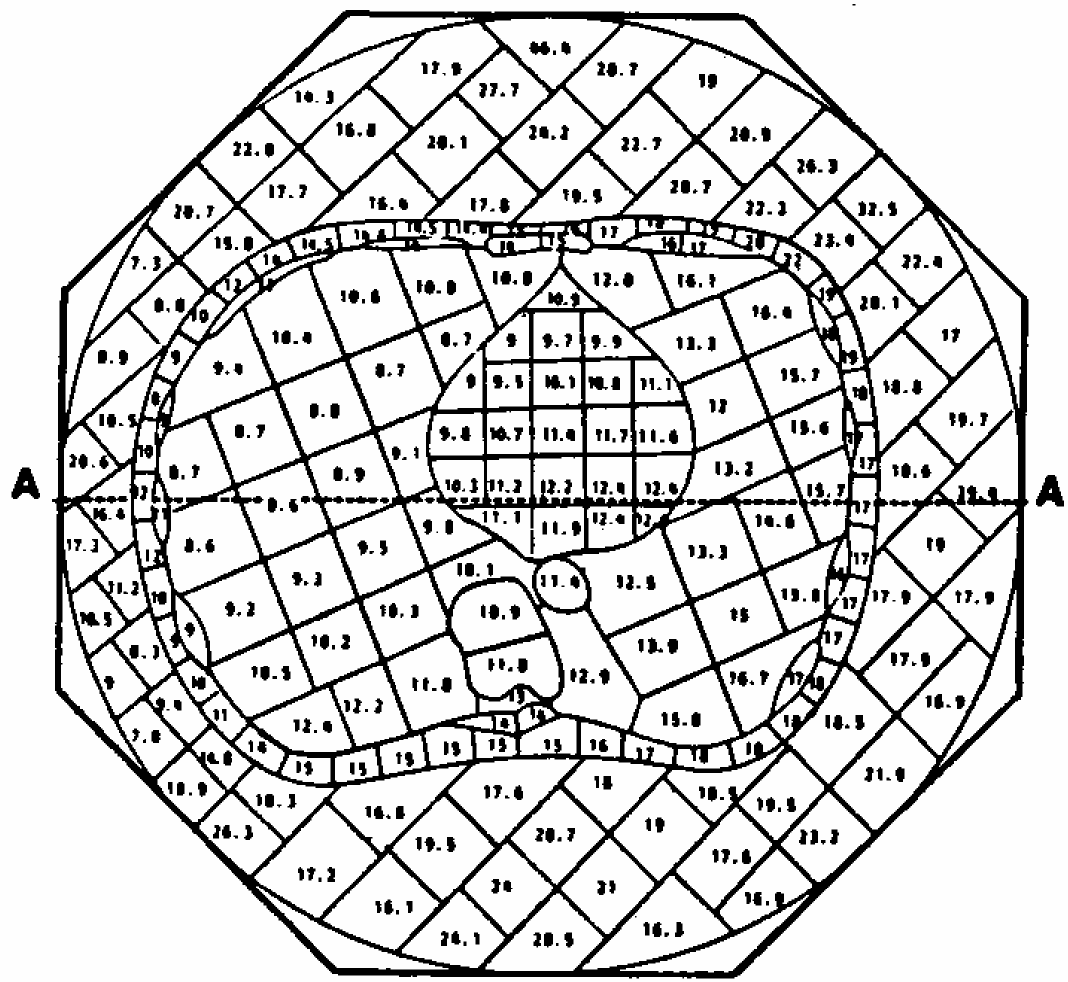
Method of Moments simulation of the microwave power deposition in human body

- Series of papers on medical applications of Electromagnetics techniques:
  - Cancer treatment
  - Measurement of lung water contents
  - Vital signs measurements



Observations:

- Many important engineering problems are formulated in terms of Integral equations, including:
  - Antenna design and characterization
  - EM scattering and absorption
  - This in addition to transmission lines which could also be alternatively formulated in terms of integral equation
- Method of Moments (MoM) represents an attractive procedure for solving this kind of open boundary (radiation and scattering-type) problems.
- We will introduce MoM by formulating and solving 1D electrostatic problem, then will discuss and solve a 2D case
- For antennas and scattering problems, we will formulate the desired integral equations, and then apply the MoM solution procedure to specific geometries and discuss the results.
- To help assess the understanding of this method, one or two assignments which include programming will be given to students

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