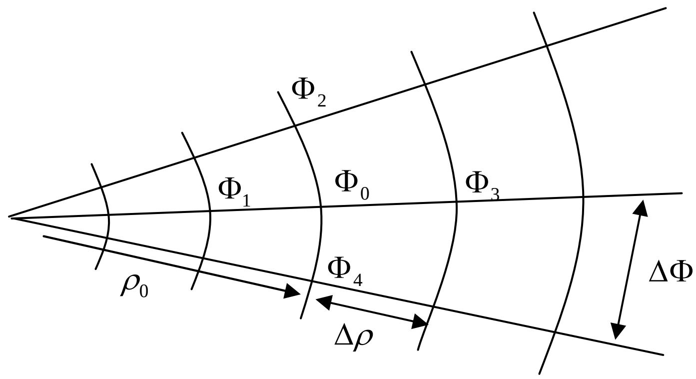
<u>FINITE DIFFERENCE IN CYLINDRICAL COORDINATES</u>

As indicated earlier, some of the nonrectangular boundaries may be represented approximately using the un-equal arm finite difference equation. This may or may not give accurate solutions depending on the boundary conditions. If the quantity of interest, say the ___ potential Φ is zero at the boundary, the un-equal arm star finite difference method would give good approximate results. If Φ is to satisfy the Neumann (zero derivative boundary condition) on the other hand, the steps–like change in boundaries may result in singular behavior of Φ at the corners, and hence unacceptable errors in the solution. For these cases, alternative finite difference representatives should be very helpful. Let us try a finite difference representation for circular regions.

$\nabla^2\Phi$ in the 2D polar coordinates is given by $\nabla^2\Phi = \frac{\partial^2\Phi}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\Phi}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2}$ where (ρ,Φ) are the cylindrical coordinates

Using the central difference



$$\frac{\partial^2\Phi}{\partial\rho^2} = \frac{\Phi_3+\Phi_1-2\Phi_0}{(\Delta\rho)^2}$$

$$\frac{1}{\rho}\frac{\partial\Phi}{\rho} = \frac{1}{\rho_0}\frac{\Phi_3-\Phi_1}{2(\Delta\rho)}$$

$$\frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2} = \frac{1}{\rho_0^2}\frac{\Phi_2+\Phi_4-2\Phi_0}{(\Delta\Phi)^2}$$

The difference equation in the cylindrical (polar) coordinates is then given by

$$\nabla^2\Phi = \frac{\Phi_3+\Phi_1-2\Phi_0}{(\Delta\rho)^2} + \frac{1}{\rho_0}\frac{\Phi_3-\Phi_1}{2(\Delta\rho)} + \frac{1}{\rho_0^2}\frac{\Phi_2+\Phi_4-2\Phi_0}{(\Delta\Phi)^2}$$

Rearranging the terms, we obtain

$$\nabla^2\Phi = \frac{1}{(\Delta\rho)^2}\left[\left(1-\frac{\Delta\rho}{2\rho_0}\right)\Phi_1 + \left(1+\frac{\Delta\rho}{2\rho_0}\right)\Phi_3 + \left(\frac{\Delta\rho}{\rho_0\Delta\Phi}\right)^2\Phi_2 + \left(\frac{\Delta\rho}{\rho_0\Delta\Phi}\right)^2\Phi_4 - \left(2+2\left(\frac{\Delta\rho}{\rho_0\Delta\Phi}\right)^2\right)\Phi_0\right]$$

Application of this difference equation will also result in a sparse matrix that lends itself to iterative methods of solution.